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## **Comportamento estrutural de ligações Viga-Pilar com chapa de topo**

*Structural Behavior of End-Plate Beam-to-Column Connections*

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### **Resumo**

A rigidez das ligações viga-pilar em estruturas de aço desempenha um papel crítico na segurança e no desempenho estrutural. Este estudo apresenta uma revisão abrangente sobre a classificação, modelagem e dimensionamento de ligações metálicas, com base em normas internacionais como a AISC 360-22 e o Eurocode 3. Dá-se ênfase especial à influência dos componentes da ligação na rigidez rotacional e às implicações para a análise estrutural. O estudo também aplica os procedimentos descritos no AISC Design Guide 16 para avaliar diferentes configurações de chapas de topo. Embora sejam discutidos modelos teóricos, o trabalho não realiza o cálculo direto das rotações nas ligações, incentivando os projetistas a avaliarem-nas por meio de métodos apropriados.

**Palavras-chave:** Rigidez rotacional. Ligação chapa de topo. Ligações viga-pilar. Estruturas metálicas. Ligações semirrígidas.

### **Abstract**

The stiffness of beam-to-column connections in steel structures plays a critical role in ensuring structural safety and performance. This study presents a comprehensive review of the classification, modeling, and design of steel connections, based on international standards such as AISC 360-22 and Eurocode 3. Particular emphasis is placed on the influence of connection components on rotational stiffness and the implications for structural analysis. The study also applies the procedures outlined in the AISC Design Guide 16 to evaluate different end-plate configurations. While theoretical models are discussed, the study refrains from calculating actual connection rotations, encouraging designers to assess them through appropriate methods.

**Keywords:** Rotational Stiffness. End-Plate Connections. Beam-to-Column Joints. Steel Frame Structures. Semi-Rigid Connections.

## **1. Introduction**

Conventional steel structures for industrial use are construction systems composed predominantly of steel members, designed to support equipment, organize production facilities, or serve as operational and control buildings. They are widely used across various industrial sectors due to their durability, strength, cost-effectiveness, time of fabrication, and constructability and maintainability. The main components of steel structures include columns, beams, girders, bracings, and trusses, which are primarily connected through welding, connection plates, and bolts. The connection between components is responsible for transferring internal forces from one member to another.

The stiffness of connections in steel structures is a crucial aspect for ensuring the integrity and efficiency of these constructions. When choosing between a rigid or pinned connection in a steel structure, it is essential to consider several factors that influence the connection's stiffness. Stiffness determines how forces are transmitted between components, directly affecting the structure's stability and load-bearing capacity. Therefore, understanding the key aspects of the stiffness of beam-to-column connections is essential to ensure structural safety and functionality. This study explores the

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relevance of connection stiffness between elements of a steel structure, providing structural engineers with greater expertise when defining the connections in their designs.

To provide a theoretical foundation for this study, a literature review was conducted on the stiffness of connections in steel structures, following the guidelines of the American Institute of Steel Construction (AISC) Design Guide 16, among other references. The types of connections (rigid, pinned, and semi-rigid) are defined, and the main factors influencing connection stiffness are discussed. The adopted methodology includes the use of normative parameters described by the AISC, which cover the calculation of stiffeners, flange lever arms, end plates, and bolts, in accordance with the design guidelines of the referenced standards.

## 2. Theoretical Foundation

The analysis of connections in steel structures is essential to ensure the safety and efficiency of buildings. Beam-to-column joints play a crucial role in the distribution of internal forces and in the overall stability of the structure. According to the AISC A360-22 – Specification for Structural Steel Buildings, connections must be designed using the Load and Resistance Factor Design (LRFD) method, ensuring that structures meet safety and performance requirements.

Connections can be classified as rigid, pinned, or semi-rigid, depending on their ability to maintain the original angle between connected members. A connection is considered rigid if, after loading, it retains 90% or more of the stiffness required to maintain the angle unchanged, AISC (2017, Steel Construction Manual). Pinned connections allow rotation between connected elements, while semi-rigid connections exhibit intermediate behavior. According to the AISC Steel Construction Manual, 15th Edition, semi-rigid connections have sufficient stiffness to influence the moment distribution in the structure, AISC (2017, Steel Construction Manual).

Analytical modeling of connections is fundamental to predict their behavior under different loading conditions. Various theoretical, empirical, and semi-empirical models are used, including the polynomial model by Frye and Morris (1975), which employs polynomial functions to describe the moment–rotation relationship of connections. This model is widely adopted due to its simplicity and accuracy in representing connection behavior, Prabha et al. (2015).

$$M = a_0 + a_1\theta + a_2\theta^2 + a_3\theta^3 + \dots + a_n\theta^n \quad (\text{Eq. 1})$$

Where:

$M$ : applied moment at the connection,

$\theta$ : connection rotation,

$a_0, a_1, a_2, \dots, a_n$  empirically determined coefficients.

This model is applied to various types of connections, such as single-web angle (SWA),

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double-web angle (DWA), top and seat angle without web angle (TS), end-plate with and without column stiffeners (EEC and EEP), header plate (HP), and T-stub connections (Ts), Patnana et al. (2019).

The accuracy and simplicity of the Frye and Morris polynomial model are the main reasons for its widespread acceptance in the structural engineering community. However, it is important to note that the model may overestimate or underestimate connection stiffness depending on specific assembly conditions and test parameters, Sommer (1980).

Studies indicate that accounting for connection deformability can lead to more economical and realistic designs. The AISC Design Guide 16 also emphasizes the importance of considering flange lever arm effects and the strength of bolts and end plates in connection design, AISC (2017, Steel Construction Manual). Walter Pfeil, in his book *Estruturas de Aço – Dimensionamento Prático*, highlights that proper evaluation of connection stiffness is essential to ensure the safety and efficiency of steel structures, Pfeil (2017).

The practical application of theoretical models is illustrated through examples and discussions. The analysis of simple and multi-story planar frames demonstrates how connection stiffness affects lateral displacements and internal force distribution. These examples are supported by the AISC Design Guide 16, which provides detailed guidelines for connection design and verification, AISC (2017, Steel Construction Manual).

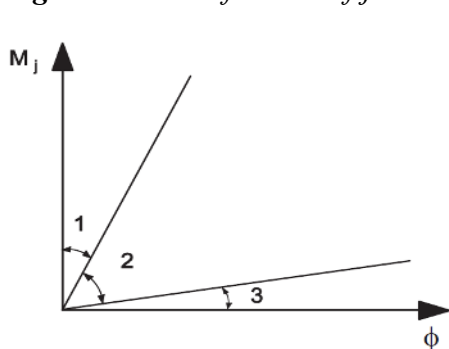
Connection elements and components must be designed to satisfy the applicable strength limit states, ensuring that their design resistance is equal to or greater than the required strength. The required strength should be determined from structural analysis under factored load combinations, or, in some cases, taken as a minimum predefined value or as a percentage of the resistance of one of the connected members, Facury (2015).

Technical literature presents different classification systems that define limits based on stiffness, strength, and rotational capacity criteria, which are widely adopted in the technical-scientific community. Notably, the classification criteria established by the European standard EN 1993-1-8:2010 define connection categories based on stiffness and strength.

According to EN 1993-1-8:2010, in terms of stiffness, connections can be classified as:

- Rigid connections: those that possess sufficient rotational stiffness to justify analysis based on full continuity.
- Pinned connections: those capable of transmitting internal forces without developing significant moments that could adversely affect the connected components or the structure as a whole.
- Semi-rigid connections: those that do not meet the criteria for either rigid or pinned connections.

**Figure 1 - Classification of joints by stiffness CEN, (2010)**



Zone 1: rigid, if  $S_{j,ini} \geq k_b EI_b / L_b$

where:

$k_b = 8$  for frames where the bracing system reduces the horizontal displacement by at least 80 %

$k_b = 25$  for other frames, provided that in every storey  $K_b/K_c \geq 0,1$  <sup>\*)</sup>

Zone 2: semi-rigid

All joints in zone 2 should be classified as semi-rigid. Joints in zones 1 or 3 may optionally also be treated as semi-rigid.

Zone 3: nominally pinned, if  $S_{j,ini} \leq 0,5 EI_b / L_b$

<sup>\*)</sup> For frames where  $K_b/K_c < 0,1$  the joints should be classified as semi-rigid.

According to EN 1993-1-8:2010, in terms of strength, connections can be classified as:

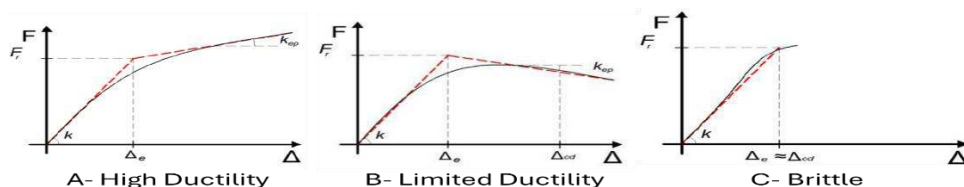
- Full-strength connections: those in which the design strength of the connection is equal to or greater than the strength of the connected members. In this case, the plastic hinge develops in the member rather than in the connection.

- Nominally pinned connections: those capable of transmitting internal forces without developing significant moments that could affect the members or the structure as a whole. These connections must also have sufficient rotational capacity to accommodate the rotations resulting from the applied loads.

- Partial-strength connections: those that do not meet the criteria for either full-strength or nominally pinned connections.

Once the connection has been conceptually defined, the next step is to characterize the behavior of the basic components through their respective force–displacement curves. These curves are typically nonlinear and should account for potential interactions with other components within the connection. However, they can be approximated by simpler representations, such as lineares curves or piecewise linear curves, without significant loss of accuracy, Silva (2003).

**Figure 2 - Behavior of high ductility, limited ductility and brittle connections, Silva (2003)**

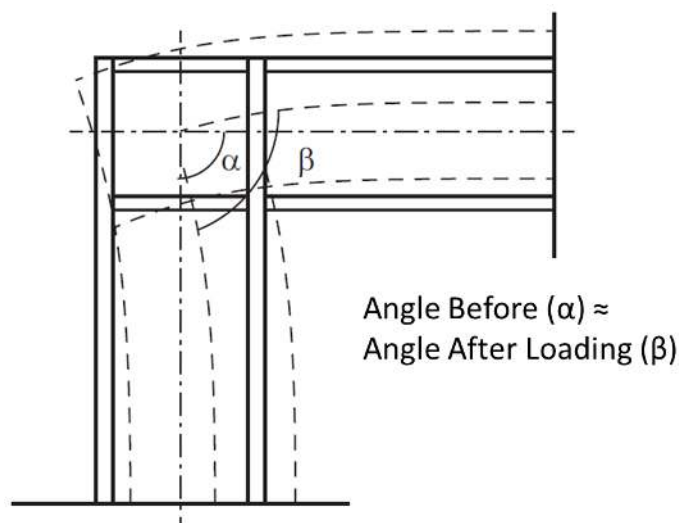


The characterization of the force–displacement curve shown in Figure 2 generally results from extensive research efforts, combining experimental data with numerical simulations based on the finite element method, in order to calibrate simplified analytical models. For each basic component, a design resistance ( $F_r$ ), a displacement limit ( $\Delta_e$ ), and a translational stiffness ( $k$ ) are defined, which are necessary to establish the representative force–displacement curve. In general,

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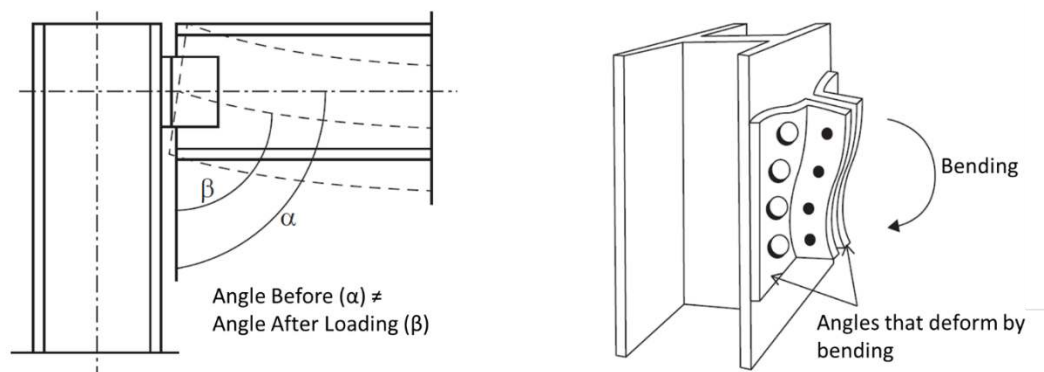
components can be classified into three categories: high ductility components, limited ductility components, and brittle components, Silva (2003).

**Figure 3 - Relative rotation of rigid connection, Facury (2015)**



In a rigid connection, the angle between the intersecting members remains unchanged after the structure is loaded (Figure 3), even under high bending moments. In this type of connection, it is assumed that there is full transmission of bending moment, shear force, and axial force between the connected structural components, Facury (2015).

**Figure 4 - Relative rotation of pinned connection, Facury (2015)**



According to Facury (2015), in pinned connections, the relative rotation between intersecting members can vary significantly. Although the transmitted moment is minimal, there is full transmission of shear force, and axial force transmission may also occur. Figure 4 illustrates the behavior of a pinned beam-to-column connection using bolts and angle brackets, where the deformation of the angles is the primary factor enabling rotation.

It is essential to consider the guidelines and recommendations of widely adopted standards, such as AISC 360-22 and Eurocode 3 Part 1.8, which provide a solid foundation for the design and analysis of steel structures. High-strength bolts are primarily used in heavy structures, particularly

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where major joints are subjected to large loads or dynamic forces. According to AISC 360-22, ASTM A325 bolts are recommended for such applications, offering minimum tensile strengths of 120 ksi (830 MPa) and 150 ksi (1040 MPa), depending on the category, AISC (2022, ANSI/AISC 360-22 Specification). In the European context, Eurocode 3 Part 1.8 recommends the use of class 8.8 bolts, which are similar to ASTM A325 bolts but exhibit slightly different yield and ultimate strengths CEN, (2010).

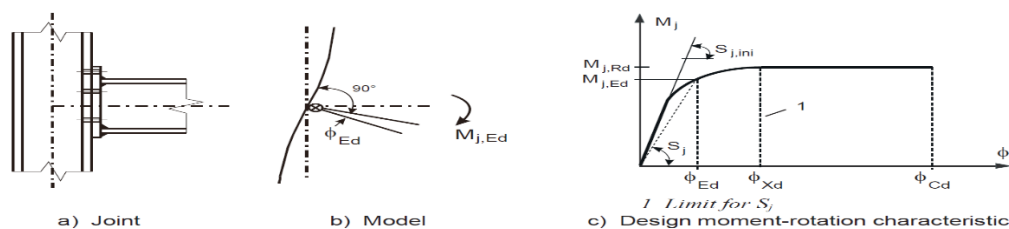
High-strength bolts can be applied in two main ways. When a connection is required to prevent any relative movement between the connected plates, the bolts must be designed with a safety factor against slip. These are referred to as friction-type connections, which exhibit higher mechanical resistance due to the friction developed between the bolt and the plate surfaces. Alternatively, bearing-type connections - also referred to as bearing or contact connections - permit limited slip and are designed to transfer loads primarily through direct bearing between the connected elements, Pfeil (2017).

In friction-type connections, bolt pretensioning is required and can be achieved using a calibrated torque wrench or the turn-of-nut method, as accepted by NBR 8800 (2008) Bellei (2010). In contrast, bearing-type connections do not require initial pretensioning and can be assembled using standard tightening procedures, Pfeil (2017).

The EN 1993-1-8:2010 standard classifies connections as rigid, semi-rigid, or nominally pinned, based on their initial rotational stiffness,  $S_{j,ini}$ . Rigid connections possess sufficient stiffness to justify structural analysis based on full continuity. Semi-rigid connections exhibit predictable interaction between connected elements, influencing the distribution of moments within the structure. Nominally pinned connections are capable of transmitting internal forces without developing significant moments that could adversely affect the connected components or the structure as a Whole, CEN 2010.

The resistance of a connection must be determined based on the strength of its basic components. The standard allows for either linear-elastic or elastic-plastic analysis in the design of connections. For example, the resistance of a beam-to-column connection can be derived from the internal force distribution and the strength of its basic components, such as the web of the column in compression and tension, and the bending resistance of the end plate, CEN (2010).

**Figure 5 - Design moment-rotation characteristic for a joint CEN (2010)**



To model the deformational behavior of a beam-to-column connection, it is necessary to consider both the shear deformation of the column web and the rotational deformation of the connection. The standard suggests modeling the connection as a rotational spring, linking the centerlines of the connected members at their intersection point. The moment-rotation characteristic of the connection must define its moment resistance, rotational stiffness, and rotation capacity, and is generally nonlinear, CEN (2010).

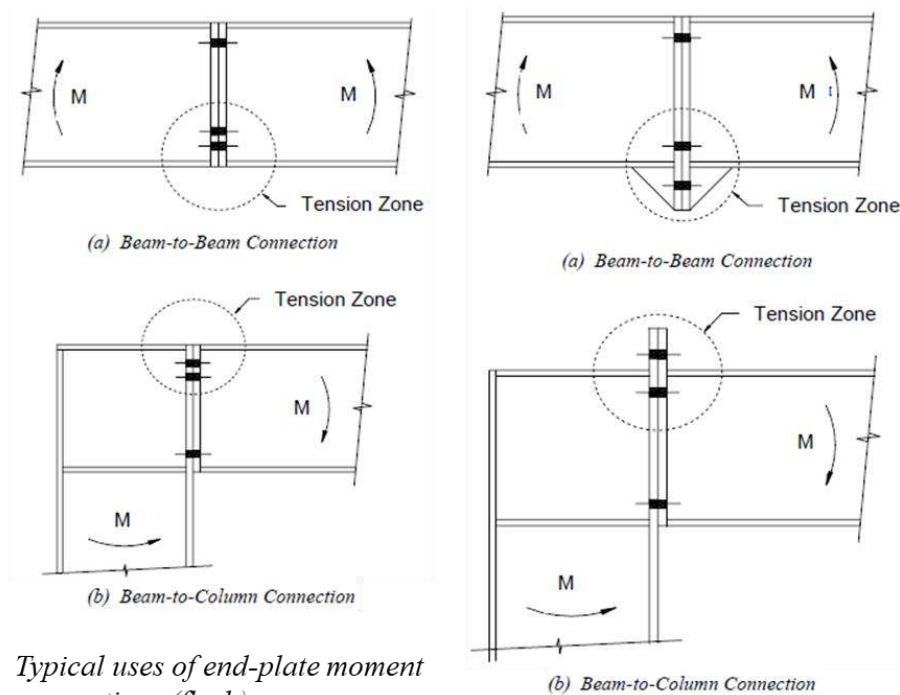
Following the analysis of the guidelines provided by EN 1993-1-8:2010, which offer a solid foundation for the design and classification of joints in steel structures, it is also important to consider the recommendations of the Steel Design Guide Series 16. This guide complements the European approach by providing specific methods for calculating the rotation between connected members in moment-resisting connections, in accordance with the AISC Specification for Structural Steel Buildings.

### 3. Methodology

Moment end-plate connections have been widely used in the low-rise steel construction industry in the United States, particularly for connecting beams to columns and beam segments in typical gable frame structures, Murray (2002). These connections are classified as fully restrained (FR) or Type 1, as specified in the AISC Load and Resistance Factor Design (LRFD) Specification and the AISC Allowable Stress Design (ASD) Specification. A typical moment end-plate connection consists of a steel plate welded to the end of a beam section and fastened to an adjacent member using rows of high-strength bolts, Murray (2002).

Moment end-plate connections are further categorized as flush or extended, with or without stiffeners, and are also classified based on the number of bolts in the tension flange. Depending on the direction of the moment and the possibility of moment reversal, the bolted end plate may be designed to resist tension in the top flange, bottom flange, or both. A flush connection is detailed such that the end plate does not extend significantly beyond the beam flanges, with all bolts located between the flanges. In contrast, an extended connection projects beyond the tension flange far enough to allow bolt placement outside the beam flanges, Murray (2002).

**Figure 6 - Typical uses of end-plate moment connections, Murray (2002)**



*Typical uses of end-plate moment connections (flush).*

*Typical uses of end-plate moment connections (extended).*

According to the AISC Design Guide 16, the rotation between connected members in moment-resisting connections is a critical factor in the design of steel structures. The ability of a connection to restrain rotation is influenced by several factors, including the stiffness of its components, such as bolts, end plates, and stiffeners.

To evaluate whether a connection can be classified as a Type I, Fully Restrained (FR) Moment Connection, the guide establishes that the maximum rotation allowed must not exceed 10% of the rotation of a simply supported beam under the same loading conditions. This reference rotation can be estimated using the following expression:

$$\theta_{Sol} = \frac{M_{Sol} \times L}{2 \times E \times I} \quad (\text{Eq. 2})$$

Where:

$M_{Sol}$ : is the applied bending moment,

$L$ : is the span length of the beam,

$E$ : is the modulus of elasticity of the beam material,

$I$ : is the moment of inertia of the beam cross-section.

This equation provides a theoretical upper limit for the rotation of a simply supported beam under a given moment. The admissible rotation for a Type I connection is then defined as 10% of this value, serving as a benchmark for evaluating the rotational capacity of the connection.

The Design Guide 16 also presents a detailed flowchart for the design of bolted end-plate connections, guiding the verification and selection process for the appropriate connection type. This

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flowchart begins with the definition of applied loads and member geometry, allowing the classification of the connection as either a flush end-plate or an extended end-plate. Based on this classification, essential parameters such as bolt number and layout, and end-plate thickness are determined to ensure compliance with structural requirements, Murray (2002).

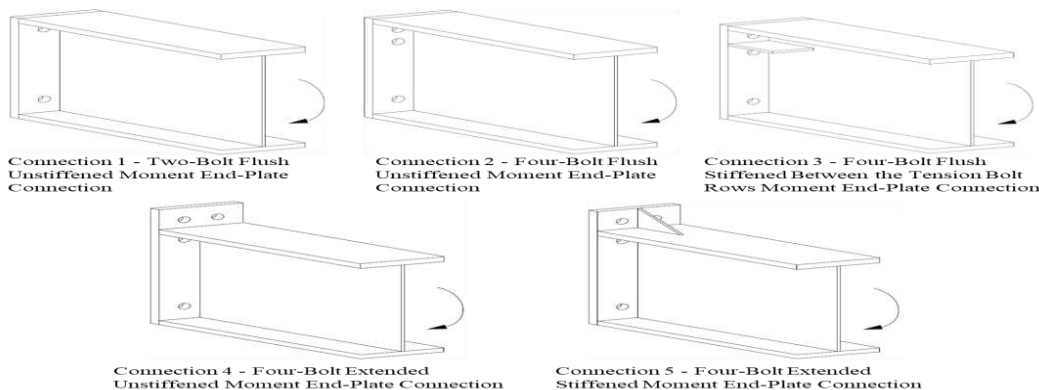
After selecting the appropriate connection type, the design process proceeds with the verification of the connection's strength, considering the applied bending moment and potential failure modes. The final design is then refined based on the results, ensuring that the end-plate thickness and bolt diameter comply with the applicable code requirements. This structured methodology ensures an efficient design aligned with the AISC guidelines.

Design Procedure 2 from Murray (2002) outlines a systematic method for designing bolted end-plate connections, ensuring that structural parameters meet both strength and stiffness criteria. The procedure begins by evaluating the applied loads and the geometry of the connected members, allowing for the definition of key connection parameters such as bolt layout and end-plate thickness. The connection's rotational performance is then assessed based on the maximum allowable rotation, according to the recommendations of the AISC Design Guide 16, which limits the rotation of a Type I, Fully Restrained (FR) Moment Connection to 10% of the rotation of a simply supported beam under the same loading conditions.

A comparative analysis of Design Procedures 1 and 2, as presented in the Design Guide 16, shows that both can result in connections with equivalent stiffness when properly designed. Design Procedure 1 typically uses thicker end plates and smaller-diameter bolts, while Design Procedure 2 employs thinner plates and larger-diameter bolts. In Procedure 2, the connection stiffness is ensured by the bolts' ability to resist tensile forces, including prying action, which can lead to a more economical design due to the reduced plate thickness. Therefore, for this study, Design Procedure 2 is adopted, following the guidance of the Design Guide 16, for its cost-effectiveness.

Based on the methodology described above, the following section presents the calculation procedures and comparative analysis of five different end-plate connection configurations.

**Figure 7 - Isometric view of the five end-plate connection configurations analyzed in this study**



### Input Data for the Calculation of End-Plate Connections:

Required factored moment ( $M_u$ ):	30 kN.m
Beam and plate material:	ASTM A572 Gr. 50
Yield stress ( $F_y$ )( $F_{py}$ ):	345 MPa
Modulus of elasticity ( $E$ ):	200 GPa
End-plate width ( $b_p$ ):	146.0 mm
Total beam depth ( $h$ ):	258.0 mm
Beam flange width ( $b_f$ ):	146.0 mm
Beam web thickness ( $t_w$ ):	6.1 mm
Beam flange thickness ( $t_f$ ):	9.1 mm
Plate thickness ( $t_p$ ):	12.5 mm
Bolt diameter ( $d_b$ ):	16 mm
Bolt standard:	A325M
Nominal tensile strength of bolts ( $F_t$ ):	$0.75 \cdot 830 \text{ MPa} = 622.5 \text{ MPa}$
Bolt gage ( $g$ ):	86 mm
Distance from the bolt centerline to the near face ( $p_s$ ):	30 mm
Load factor to limit connection rotation at ultimate moment to 10% of simple span rotation ( $y_r$ ):	1.25 for flush connection 1.00 for extended connection
Resistance factor for end-plate yield ( $\phi_b$ ):	0.9
Bolt pretension force according Design Guide 16 recommendation ( $T_b$ ):	68.25 kN

### The key equations used in the design calculations are listed below:

$$s = \frac{1}{2} \cdot \sqrt{b_p \cdot g} \quad (\text{Eq. 3})$$

$$P_t = \pi \cdot d_b^2 \cdot \frac{F_t}{4} \quad (\text{Eq. 4})$$

$$t_{p.req} = \sqrt{\frac{y_r \cdot M_u}{\phi_b \cdot f_y \cdot Y}} \quad (\text{Eq. 5})$$

$$a_i = 3.682 \cdot \left(\frac{t_p}{d_b}\right)^3 - 0.085 \text{ in} \quad (\text{Eq. 6})$$

$$a_o = 3.682 \cdot \left(\frac{t_p}{d_b}\right)^3 - 0.085 \text{ in} \leq (p_{ext} - p_{f,o}) \quad (\text{Eq. 7})$$

$$w' = \frac{b_d}{2} - \left(b_d + \frac{1}{16} \text{ in}\right) \quad (\text{Eq. 8})$$

$$F'_i = \frac{t_p^2 \cdot F_{py} \cdot \left(0.85 \cdot \frac{b_d}{2} + 0.80w'\right) + \frac{\pi \cdot b_d^3 \cdot F_t}{8}}{4 \cdot P_{f,i}} \quad (\text{Eq. 9})$$

$$F'_o = \frac{t_p^2 \cdot F_{py} \cdot \left(0.85 \cdot \frac{b_d}{2} + 0.80w'\right) + \frac{\pi \cdot b_d^3 \cdot F_t}{8}}{4 \cdot P_{f,o}} \quad (\text{Eq. 10})$$

$$Q_{max,i} = \frac{w' \cdot t_p^2}{4 \cdot a_i} \cdot \sqrt{F_{py}^2 - 3 \cdot \left(\frac{F'_i}{w' \cdot t_p}\right)^2} \quad (\text{Eq. 11})$$

$$Q_{max,o} = \frac{w' \cdot t_p^2}{4 \cdot a_o} \cdot \sqrt{F_{py}^2 - 3 \cdot \left(\frac{F'_o}{w' \cdot t_p}\right)^2} \quad (\text{Eq. 12})$$

$$\phi_b M_{pl} = \phi_b \cdot F_{py} \cdot t_p^2 \cdot Y \quad (\text{Eq. 13})$$

$$Y = \frac{b_p}{2} \left[ h_1 \left( \frac{1}{p_f} + \frac{1}{s} \right) \right] + \frac{2}{g} [h_1(p_f + s)] \quad (\text{Eq. 14})$$

$$Y = \frac{b_p}{2} \left[ \left( \frac{h_1}{p_f} \right) + \left( \frac{h_2}{s} \right) \right] + \frac{2}{g} [h_1(p_f + 0.75p_b) + h_2(s + 0.25p_b)] + \frac{g}{2} \quad (\text{Eq. 15})$$

$$Y = \frac{b_p}{2} \left[ \left( \frac{h_1}{p_f} + \frac{h_1}{p_{s,o}} \right) + \left( \frac{h_2}{s} + \frac{h_2}{p_{s,i}} \right) \right] + \frac{2}{g} [h_1(p_f + p_{s,o}) + h_2(s + p_{s,i})] \quad (\text{Eq. 16})$$

$$Y = \frac{b_p}{2} \left[ \left( \frac{h_1}{p_{f,i}} + \frac{h_1}{s} \right) + \left( \frac{h_o}{p_{f,o}} \right) - \frac{1}{2} \right] + \frac{2}{g} [h_1(p_{f,i} + s)] \quad (\text{Eq. 17})$$

$$Y = \frac{b_p}{2} \left[ \left( \frac{h_1}{p_{f,i}} + \frac{h_1}{s} \right) + \left( \frac{h_o}{p_{f,o}} + \frac{h_o}{2s} \right) - \frac{1}{2} \right] + \frac{2}{g} [h_1(p_{f,i} + s) + h_o(d_e + p_{f,o})] \quad (\text{Eq. 18})$$

$$\phi M_q = \max \left\{ \begin{array}{l} 0.75 \cdot [2 \cdot (P_t - Q_{max,i}) \cdot d_1] \\ 0.75 \cdot [2 \cdot T_b \cdot d_1] \end{array} \right. \quad (\text{Eq. 19})$$

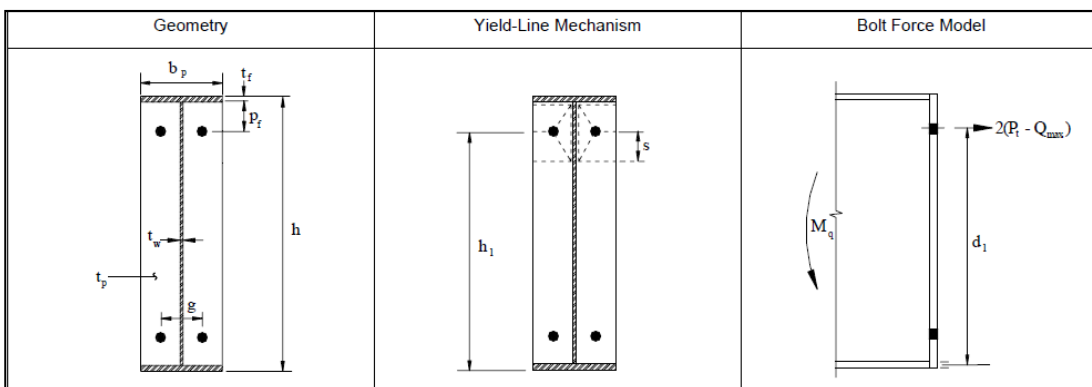
$$\phi M_q = \max \left\{ \begin{array}{l} 0.75 \cdot [2 \cdot (P_t - Q_{max,i}) \cdot (d_1 + d_2)] \\ 0.75 \cdot [2 \cdot T_b \cdot (d_1 + d_2)] \end{array} \right. \quad (\text{Eq. 20})$$

$$\phi M_q = \max \left\{ \begin{array}{l} 0.75 \cdot [2 \cdot (P_t - Q_{max,o}) \cdot d_0 + 2 \cdot (P_t - Q_{max,i}) \cdot d_1] \\ 0.75 \cdot [2 \cdot (P_t - Q_{max,o}) \cdot d_0 + 2 \cdot T_b \cdot d_1] \\ 0.75 \cdot [2 \cdot (P_t - Q_{max,i}) \cdot d_1 + 2 \cdot T_b \cdot d_0] \\ 0.75 \cdot [2 \cdot T_b \cdot (d_0 + d_1)] \end{array} \right. \quad (\text{Eq. 21})$$

### Connection 1 - Two-Bolt Flush Unstiffened End-Plate

This configuration uses two rows of bolts placed between the beam flanges, with no stiffeners. It represents a basic moment-resisting connection.

**Figure 8 - Summary of Two-Bolt Flush Unstiffened Moment End-Plate Analysis, Murray (2002)**



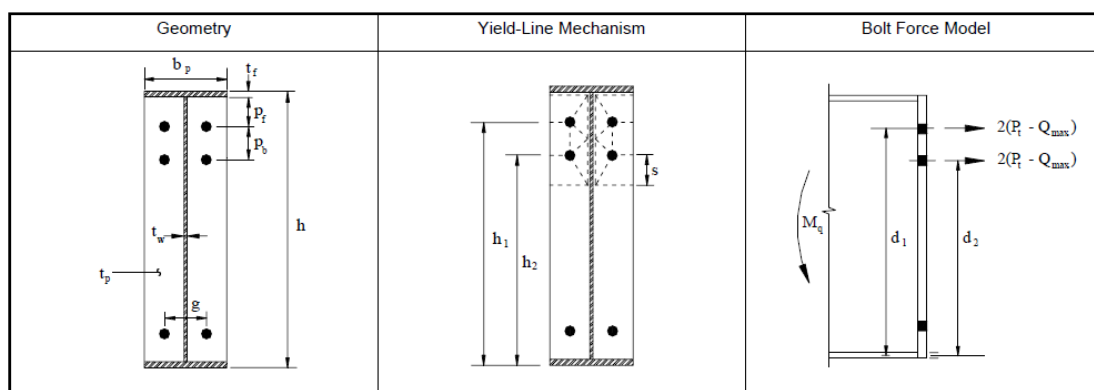
**Table 1 - Two-Bolt Flush Unstiffened Moment End-Plate Connection Analysis Results**

$s$ (Eq. 3)	56.03 mm	$w'$ (Eq. 8)	55.41 mm
$P_t$ (Eq. 4)	125.16 kN	$F'_i$ (Eq. 9)	56.13 kN
$Y$ (Eq. 14)	1255.81 mm	$Q_{max,i}$ (Eq. 11)	16.08 kN
$t_{p,req}$ (Eq. 5)	9.81 mm	$\phi M_q$ (Eq. 19)	35.07 kN · m
$a_i$ (Eq. 6)	42.44 mm	$\phi_b M_{pl}$ (Eq. 13)	60.93 kN · m

### Connection 2 - Four-Bolt Flush Unstiffened End-Plate

This configuration increases the number of bolts to four, still without stiffeners, aiming to improve moment capacity.

**Figure 9 - Summary of Four-Bolt Flush Unstiffened Moment End-Plate Analysis, Murray (2002)**



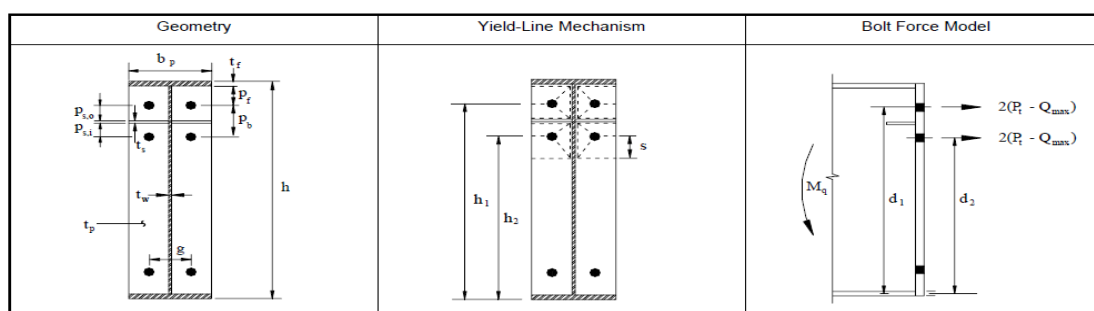
**Table 2 - Four-Bolt Flush Unstiffened Moment End-Plate Connection Analysis Results**

$s$ (Eq. 3)	56.03 mm	$w'$ (Eq. 8)	55.41 mm
$P_t$ (Eq. 4)	125.16 kN	$F'_i$ (Eq. 9)	56.13 kN
$Y$ (Eq. 15)	1408.51 mm	$Q_{max,i}$ (Eq. 11)	16.08 kN
$t_{p,req}$ (Eq. 5)	9.26 mm	$\phi M_q$ (Eq. 20)	61.97 kN · m
$a_i$ (Eq. 6)	42.44 mm	$\phi_b M_{pl}$ (Eq. 13)	68.33 kN · m

### Connection 3 - Four-Bolt Flush Stiffened End-Plate

Stiffeners are added between the tension bolt rows to enhance rotational stiffness and reduce plate deformation.

**Figure 10 - Summary of Four-Bolt Flush Stiffened Moment End-Plate Analysis, Murray (2002)**



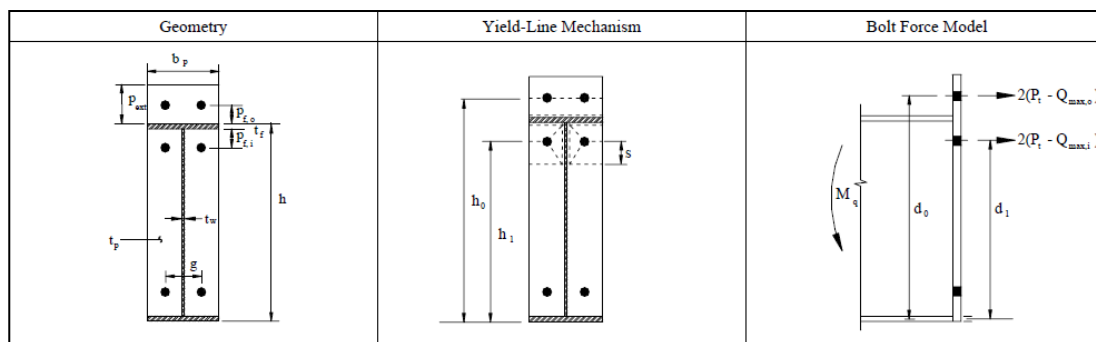
**Table 3 - Four-Bolt Flush Stiffened Moment End-Plate Connection Analysis Results**

$s$ (Eq. 3)	56.03 mm	$w'$ (Eq. 8)	55.41 mm
$P_t$ (Eq. 4)	125.16 kN	$F'_i$ (Eq. 9)	56.13 kN
$Y$ (Eq. 16)	2339.72 mm	$Q_{max,i}$ (Eq. 11)	16.08 kN
$t_{p,req}$ (Eq. 5)	7.18 mm	$\phi M_q$ (Eq. 20)	61.97 kN · m
$a_i$ (Eq. 6)	42.44 mm	$\phi_b M_{pl}$ (Eq. 13)	113.51 kN · m

**Connection 4 - Four-Bolt Extended Unstiffened End-Plate**

The end plate extends beyond the beam flange, allowing bolt placement outside the flange region, improving lever arm efficiency.

**Figure 11 - Summary of Four-Bolt Extended Unstiffened Moment End-Plate Analysis, Murray (2002)**



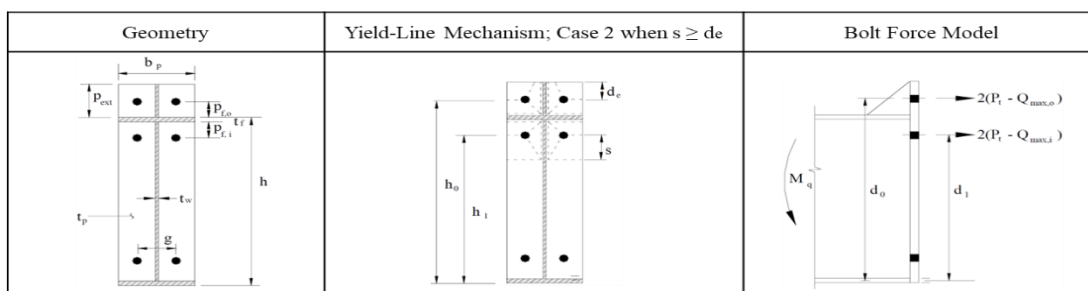
**Table 4 - Four-Bolt Extended Unstiffened Moment End-Plate Connection Analysis Results**

$s$ (Eq. 3)	56.03 mm	$F'_i$ (Eq. 9)	56.13 kN
$P_t$ (Eq. 4)	125.16 kN	$F'_o$ (Eq. 10)	56.13 kN
$Y$ (Eq. 17)	1882.93 mm	$Q_{max,i}$ (Eq. 11)	16.08 kN
$t_{p,req}$ (Eq. 5)	7.16 mm	$Q_{max,o}$ (Eq. 12)	16.08 kN
$a_i$ (Eq. 6)	42.44 mm	$\phi M_q$ (Eq. 21)	81.45 kN · m
$a_o$ (Eq. 7)	42.44 mm	$\phi_b M_{pl}$ (Eq. 13)	91.35 kN · m
$w'$ (Eq. 8)	55.41 mm		

**Connection 5 - Four-Bolt Extended Stiffened End-Plate**

Combines the benefits of an extended plate and stiffeners, offering the highest strength and stiffness among the configurations analyzed.

**Figure 12 - Summary of Four-Bolt Extended Stiffened Moment End-Plate Analysis, Murray (2002)**



**Table 4 - Four-Bolt Extended Stiffened Moment End-Plate Connection Analysis Results**

$s$ (Eq. 3)	56.03 mm	$F'_i$ (Eq. 9)	56.13 kN
$P_t$ (Eq. 4)	125.16 kN	$F'_o$ (Eq. 10)	56.13 kN
$Y$ (Eq. 18)	2463.11 mm	$Q_{max,i}$ (Eq. 11)	16.08 kN
$t_{p,req}$ (Eq. 5)	6.26 mm	$Q_{max,o}$ (Eq. 12)	16.08 kN
$a_i$ (Eq. 6)	42.44 mm	$\phi M_q$ (Eq. 21)	81.45 kN · m
$a_o$ (Eq. 7)	42.44 mm	$\phi_b M_{pl}$ (Eq. 13)	119.5 kN · m
$w'$ (Eq. 8)	55.41 mm		

To facilitate comparison between the five configurations, the key performance indicators are summarized in the following chart.

**Figure 13 - Comparative summary of required plate thickness, connection strength, and utilization ratio for the five end-plate configurations**

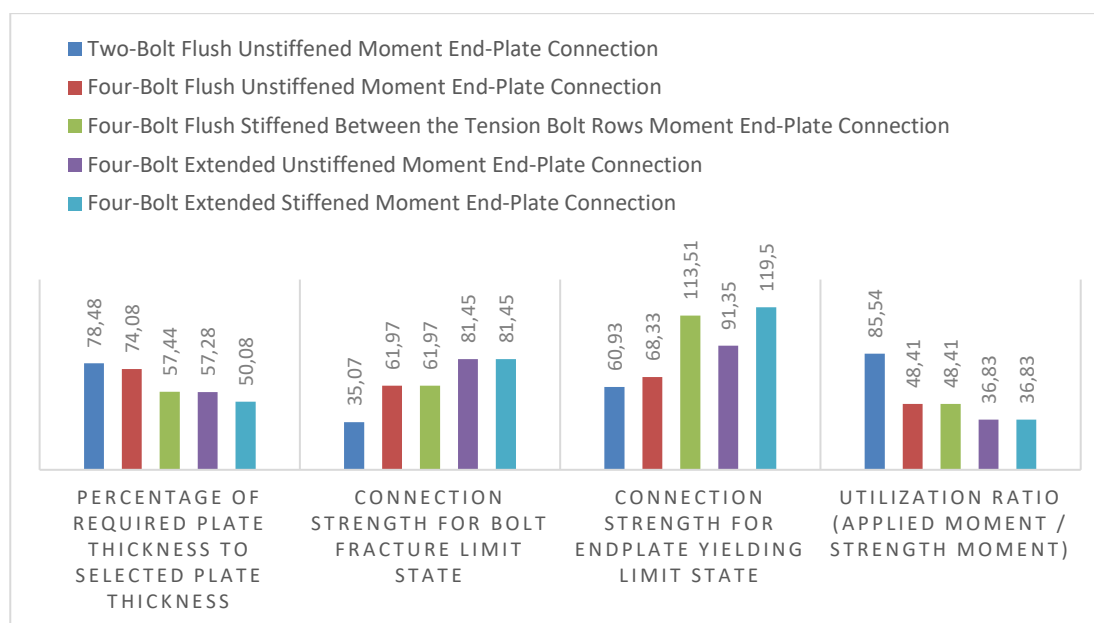


Figure 13 presents a comparative overview of the key performance indicators for the five connection configurations analyzed. The following section discusses the implications of these results and summarizes the main findings of the study.

## Conclusion

This study evaluated the structural behavior of five different end-plate beam-to-column connection configurations using the methodology outlined in the AISC Design Guide 16. The analysis considered key performance indicators, including the required plate thickness, connection strength under bolt fracture and end-plate yielding limit states, and the utilization ratio based on the applied moment.

The results demonstrate that Connection 5 – Four-Bolt Extended Stiffened End-Plate

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exhibited the highest performance in terms of strength and stiffness, achieving the greatest resistance to end-plate yielding (119,5 kN·m) and the lowest utilization ratio (36.83%), indicating a high safety margin. Similarly, Connection 4 – Four-Bolt Extended Unstiffened also performed well, particularly in bolt fracture resistance, while offering a slightly more economical alternative due to the absence of stiffeners.

In contrast, Connection 1 – Two-Bolt Flush Unstiffened showed the highest utilization ratio (85.54%) and the lowest strength values, highlighting its limited capacity for moment transfer and reduced safety margin. Although Connection 2 and Connection 3 improved upon this by increasing bolt count and adding stiffeners, respectively, their performance remained inferior to the extended configurations.

To support a more comprehensive assessment of connection efficiency, the utilization ratio was introduced as a comparative metric. This parameter enables readers to evaluate the safety margin of each configuration beyond traditional strength-based criteria, offering a practical tool for selecting cost-effective and structurally reliable solutions.

In conclusion, the findings support the adoption of Design Procedure 2 with extended and stiffened end-plates for applications requiring high moment capacity and structural reliability. These configurations not only meet the strength and stiffness requirements but also offer favorable utilization and material efficiency, aligning with the principles of safe and economical steel structure design.

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