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Application of the ordinary differential equation in the Runge-Kutta method: a systematic literature review.

Application of the ordinary differential equation in the Runge-Kutta method: a systematic literature review

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Summary

This work presents a systematic literature review on the application of the Runge-Kutta method in solving ordinary differential equations (ODEs). Ordinary differential equations are fundamental for modeling dynamic phenomena in various fields of knowledge, such as physics, biology, engineering, and social sciences. Given the difficulty of obtaining analytical solutions for many of these equations, numerical methods, especially the Runge-Kutta method, become essential tools. Thus, the systematization of knowledge about this numerical method strengthens the scientific basis and fosters new possibilities for application in strategic areas. Therefore, the present study aims to conduct a systematic literature review on the application of the Runge-Kutta method in solving ordinary differential equations, in order to identify the main contexts of use, research trends, scientific contributions, and existing gaps. To this end, a bibliographic research was employed, with a differentiated exploratory focus and a qualitative approach. Finally, the Runge-Kutta method, configured in the ordinary differential equation, offers greater ease in calculus classes, and students are able to improve their respective allusions in higher-level disciplines in the exact sciences.

Keywords: Scientific Studies, Runge-Kutta Method, Ordinary Differential Equations.

Abstract

This paper presents a systematic review of the literature on the application of the Runge-Kutta Method in solving ordinary differential equations (ODEs). Ordinary
Differential equations are fundamental for modeling dynamic phenomena in several areas of knowledge, such as physics, biology, engineering and social sciences. Given the difficulty in obtaining analytical solutions for many of these equations, numerical methods, especially the Runge-Kutta method, become essential tools. Thus, the systematization of knowledge about this numerical method strengthens the scientific basis and fosters new possibilities of application in strategic areas. Thus, the present study aims to carry out a systematic review of the literature on the application of the Runge-Kutta Method in solving ordinary differential equations, in order to identify the main contexts of use, research trends, scientific contributions and existing gaps. For this purpose, bibliographic research was used, with a differentiated exploratory and qualitative approach. Finally, the Runge-Kutta method is configured in the ordinary differential equation, providing greater facilities in calculus classes and students are able to improve their respective allusions in higher level subjects in the exact

area.

Keywords: Scientific Studies, Runge-Kutta Method, Ordinary Differential Equations.

INTRODUCTION

Theoretically, studies of the Runge-Kutta method with resolution in the equation

Ordinary differential equations (ODEs) are based on numerical mechanisms that are

extremely useful in solving many mathematical and physical problems, which are generally modeled by ordinary differential equations and emerge as an alternative for obtaining results that almost always cannot be obtained through real procedures. In turn, the The Runge-Kutta method stands out for its accuracy and applicability across a wide range of... problems, ranging from population models to control systems, electrical circuits and Nonlinear dynamics. Its relative simplicity, combined with computational efficiency, makes it... one of the most widely used methods in teaching and scientific practice. The potential of this The method includes solving complex problems with good numerical stability, in addition to... possibility of implementation in computer software widely used in science. and in engineering. The impact factor of this research lies precisely in the possibility of to consolidate existing knowledge, making it accessible to researchers, teachers and students working with mathematical modeling. In addition, it can guide new students. research and applications in areas where the use of ordinary differential equations (ODEs) It is still emerging or underexplored. The scientific relevance of the topic lies in strengthening the Theoretical and methodological basis related to the modeling and simulation of dynamic systems. The social relevance is revealed in the practical applicability of this knowledge, which can have an impact. positively in areas such as public health (epidemiological modeling), environment (pollution and climate simulation), economics (forecasting models), among others. Thus, when To contribute to the optimization and understanding of real-world processes, research promotes advancement. of knowledge and social well-being.

PROBLEM

How has the Runge-Kutta method been applied to solving equations? ordinary differentials, according to the available scientific literature, and what are the main ones? trends, contributions and gaps identified from a systematic review of literature?

OBJECTIVES

General Objective

To conduct a systematic literature review on the application of the Runge method. Kutta in solving ordinary differential equations, with the aim of identifying the main contexts of use, research trends, scientific contributions and gaps existing.

Specific Objectives

• Map the main scientific works that apply the Runge-Kutta method in
Solving ordinary differential equations, highlighting areas of knowledge and types of...
problems addressed;

• Analyze the methodological and technical contributions present in the literature on the use of
Runge-Kutta method, considering its evolution and variations (RK2, RK4);

• Identify gaps and opportunities for future research based on the results.
findings suggest new avenues for the application and improvement of the method.

JUSTIFICATION

Interest in numerical methods and their application to real-world problems motivated the The choice of this topic. The curiosity to understand how techniques like the Runge method... Kutta's methods are used in scientific practice, as well as the desire to delve deeper into the subject. knowledge of ordinary differential equations (ODEs) has driven the development of this study. Furthermore, the desire to contribute to the systematization of Technical and methodological knowledge on the subject reinforces commitment to the search.

From an academic point of view, this study is relevant because it proposes a review. A systematic approach that gathers, organizes, and critically analyzes scientific production concerning... Application of the Runge-Kutta method to ordinary differential equations (ODEs). Although Widely used, the method lacks research to solidify its application in different scientific contexts. The systematization of this knowledge not only enriches the It is based on existing literature, but also serves as a foundation for future research in the field of modeling. mathematics, teaching numerical methods and computational development.

Socially, the study is justified because it addresses a topic that has practical implications. in areas that directly impact society, such as health, environment, engineering, Economics and education. Precise modeling of real-world phenomena, using equations. Ordinary difference equations (ODEs) solved numerically contribute to the prediction, control, and... Process and system improvement. By expanding understanding of the use of the Method of Runge-Kutta, the work contributes to the training of more professionals and researchers. trained to address complex social problems. Technological advancement and the growing The complexity of contemporary scientific problems makes the use of numerical methods... increasingly necessary. The Runge-Kutta method continues to be widely used in modern computer simulations, including in emerging areas such as artificial intelligence

Artificial intelligence, analysis of complex dynamic systems, and epidemiological modeling. A
The relevance of the topic is intensified by the increase in scientific production involving it.
numerical modeling and the demands for precision in simulations.

METHODOLOGY

This research was carried out according to the following technical procedure:
bibliographic research.

Bibliographic research is the survey or review of published works on the theory.
which will guide the scientific work, requiring dedication, study, and analysis by
The researcher who will carry out the scientific work aims to gather and analyze texts.
published to support scientific work (Sousa; Oliveira, 2024).

Regarding the objective, the research can be exploratory because it provides greater...
familiarity with the problem, since it seeks to make it explicit, encompassing
Therefore, a literature review and analysis of examples that stimulate understanding were conducted.
It generally takes the form of bibliographic research and case studies (Rosa, 2024).

Regarding the research approach, it will be qualitative. Qualitative research is appropriate.
to the consideration of the dynamic relationship between the objective world and the subjectivity of the subject that
This cannot be translated into numbers. The interpretation of phenomena and the relevance of
meanings that are fundamental in the process of this analysis (Alves, 2024).

In this sense, this literature review seeks a broad reflection and in-depth analysis.
Regarding the topic, highlighting its relevance to the present day. Because it deals with
Bibliographic research: the studies conducted are based on previously published sources, which were:
Articles, dissertations, books, monographs, research papers, and periodicals.

With regard to the printed bibliography, it was examined and used by means of
from existing doctrines and books that addressed this topic, as well as through
from the internet, by searching for current web pages.

In the analysis and interpretation of the results, an analytical reading was performed with the
The purpose is to organize and summarize the information contained in the sources, so that they...
obtaining answers to the research problem.

The chosen tools and materials were read, studied, and analyzed, with the
The objective is to detect data, references, and knowledge, and to indicate relationships within the data.
acquired and evaluate the consistency of such research, as well as the veracity of each one.
of them (Pereira, 2021).

SCIENTIFIC APPLICATIONS OF THE RUNGE-KUTTA METHOD, A MAPPING BY AREAS AND THEMES

The Runge-Kutta method is probably one of the most popular methods. The fourth-order Runge-Kutta method is also one of the most valuable for obtaining approximate initial value solutions (Vera, 2022).

The Runge-Kutta method can use different coefficients or weights for the calculations. Thus, second-order (RK2) and fourth-order algorithms can be used. (RK4), for example, these being the most popular (Moreira; Schepke; Cabral, 2022).

Each Runge-Kutta method consists of comparing a Taylor polynomial. appropriate for eliminating the calculation of derivatives, by performing several evaluations of the function a each step. This is because the Runge-Kutta method can be understood as a an improvement of Euler's method, with a better estimate of the derivative of the function. (Lopes, 2021).

1.2 Scientific Sectors and Dynamic Models Addressed with the Runge-Kutta Method

The application of the Runge-Kutta method (RK-1, RK-2 and RK-4) and its efficiency in RC circuit problems, LC circuit problems, and motion in a resistive medium equations. homogeneous linear differentials with certain specific conditions, but which we must also consider. to recognize that there are other areas within physics where these principles apply. Therefore, it is It is possible to apply the Runge-Kutta method to problems involving, for example, electromagnetism, free oscillations in an RL circuit, free oscillations in an LC circuit, how in Newtonian, Lagrangian, and Hamiltonian mechanics systems, "motion in a medium" "Resistive" (Zome, 2023).

According to Zome (2023), the Runge-Kutta method for differential equations Fourth-order (RK-4) is a widely used method for computational calculation; it is in this The method we will use as a model for creating the algorithm in the Mathematica software, and Its general formulation is:

yn+1 = yn + h / 6 (k1 + 2k2 + 2k3 + k4), Numerical methods seek to develop calculation processes (algorithms) using a finite sequence of basic arithmetic operations, so that certain mathematical problems become executable. These algorithms generally involve a large number of arithmetic calculations. It is therefore not surprising that, in recent decades, with the rapid growth of the capabilities of digital computers, the role of numerical methods in solving complex problems has greatly increased (Sá, 2024).

Ordinary differential equations play an extremely important role in all areas of mathematics and physics. For this reason, the fourth-order Runge-Kutta method describes the simple pendulum with a resistive force present through its phase diagrams in subcritical, critical, and supercritical damping.

(Celeste, 2024).

1.3 Methodological Aspects of Mathematical Modeling with Runge-Kutta: RK2, RK4 and Variations

Among the many difficulties in the teaching and learning process, research has
It has been mentioned that the abstraction of the discipline of differential calculus proves to be a major challenge.
And even greater is its application and importance to other areas of knowledge.
(Fernandes; Saldanha, 2022). Regarding this importance, mathematical modeling has a
It plays a guiding role in relation to the progress of science, as its application is present in
various areas of knowledge such as physics, chemistry, biology, mathematics, among others.
In this vein, differential equations have been a great tool in solving others.
mathematical problems involving the aforementioned areas and, consequently, gained a lot
A highlight in the field of physics.

Thus, the present study, whose theme is ordinary differential equations in
mathematical modeling, which aims to highlight the importance of mathematics in the construction of
development of other natural sciences, using as a key tool the
main applications of ordinary differential equations, specifically those of the first and
Second-order modeling of physical phenomena is sparking the interest of academics.
for the importance of mathematics in scientific development (Souza, 2024).

The contributors who preceded Leonardo Euler refined his work and from there...
This produced new ideas, inaccessible to the 18th century of Euler and

Sophisticated and beyond human comprehension. The history of equations.
Differential calculus begins with the precursors of calculus: Leibniz and Newton. When these
brilliant mathematicians had an appropriate language and sufficient knowledge about
Derivatives, therefore, appeared in equations and this field of study gained notoriety.
However, they soon discovered that the solutions to these equations were not so easy.
Algebraic manipulations and simplifications helped only a little. The integral and its role.
Theorist in the fundamental theorem of calculus offered help only when the variables were
separable variables. The method of separation of variables was generalized by Leibniz, despite having
developed by Jacob Bernoulli (Boyce; Diprima, 2021). Studies of the equations
Differences have been attracting mathematicians ever since Newton began studying them.
(Boyce; Diprima, 2021).

Opportunities for Enhancing the Runge-Kutta Method in the Exact Sciences, A Looking at Mathematics and Physics

Anton, Bivens, and Davis (2024) describe that if a moving particle
A straight line has a position function *s(t),* so its velocity *v(t)* and acceleration *a(t)*

Instantaneous values are given by the equations:

$$v(t) = sÿ(t) \text{ and } a(t) = vÿ(t)$$

That is, s(t) is an antiderivative of v(t), and v(t) is an antiderivative of a(t). Thus, we have:

$$s(t) = ÿ\, v(t)dt \text{ and } v(t) = ÿ\, a(t)dt$$

Using the formula for the velocity function v(t) of a particle in rectilinear motion, then the integration of v(t) produces a family of position functions with that function velocity. If, in addition, we know the position s0 of the particle at some instant t0, then We will have enough information to find the constant of integration and determine a unique position function. Similarly, if we know the acceleration function a(t) of the particle, then integrating a(t) produces a family of velocity functions with that function acceleration. If, in addition, we know the velocity v0 of the particle at some instant t0, then We will have enough information to find the constant of integration and determine a unique velocity function (Anton; Bivens; Davis, 2024, p. 76).

Various applications of differential equations in mathematics and physics education. The applications were taken from the main calculus textbooks used in undergraduate courses. and their objective was to show that it is possible to incorporate them into the classroom context and in As a result, guide students in the pursuit of meaningful learning (Souza, 2024).

Technology has driven change in education in the 21st century and in communities. Schoolchildren began using them more frequently, especially in experiments with The study of differential calculus. In mathematics education, a process that requires strategies. and diverse teaching resources that enhance learning are essential. to understand the actions developed by teachers to improve the acquisitions of knowledge of the respective students, for the benefit of the educational field.

RESULTS AND DISCUSSION

Runge-Kutta (RK) numerical methods belong to the group of methods Iterative, that is, it is a technique that devises a sequence of approximate solutions. which improve as iterations are performed. The RK method is Possibly one of the most popular methods, and the fourth-order method is one of the most widely used. to obtain approximate initial value solutions. Each Runge-Kutta method consists of compare an appropriate Taylor polynomial to eliminate the need to calculate derivatives, by doing- if multiple function evaluations are performed at each step (Pereira, 2023).

Numerical methods are a set of applications of algorithms used to... Formulate and solve mathematical problems using arithmetic operations. Algorithms, therefore... In turn, they are finite groups of ordered operations that allow us to solve certain... Mathematical problems. This refers to a sequence of established instructions or rules that, Through these given steps, we can approximate the actual result of a given problem. (Zome, 2023).

Numerical analysis is the study of algorithms that seek numerical results from... Problems from the most diverse areas of human knowledge, modeled mathematically. (Araújo, 2022).

One of the applications of computational numerical methods is the resolution of differential equations, which play a fundamental role in all of mathematics; some, Often, we are unable to solve problems analytically; other times, we give them away. impossible (Lyra, 2018).

The fourth-order Runge-Kuta method is used to find approximate solutions to initial value. It consists of comparing a proper Taylor polynomial to eliminate the calculation. In derivatives, several evaluations of the function are performed at each step. Known as a Euler's improvement has a better standalone for the derivative of the function (Valle, 2024).

FINAL CONSIDERATIONS

The main objective of this systematic literature review was to investigate How has the Runge-Kutta method been applied in solving differential equations? ordinary data objects (EDOs), in order to identify the main contexts of use, research trends, scientific contributions and existing gaps. Based on the analysis of the works After selecting the materials, it was observed that the method is widely used in various areas. of knowledge, with emphasis on mathematics, physics, engineering, biology and sciences.

computational. The identified application contexts range from modeling to
physical systems — such as particle motion, electrical circuits, and mechanical oscillations
— even simulations in ecology, epidemiology, and population dynamics. The most significant variation
The fourth-order Runge-Kutta method (RK4) is used, given its efficient combination.
between precision and simplicity of implementation. However, adaptive and order versions
superior areas have also gained ground, especially in problems that require control of
Error management and optimization of computational resources.

In terms of scientific contributions, the literature reveals significant advances in customization and adaptation of the method for different classes of ODEs, as well as the Development of computational algorithms and software that automates their use. In addition Furthermore, there is a growing integration of the method with other approaches, such as Artificial intelligence, hybrid methods, and multidimensional simulations. However, the review It also highlighted significant gaps. In particular, there is a lack of studies that discuss... critically evaluate the choice of numerical method in relation to the specific nature of the problem. modeled, as well as the scarcity of publications aimed at teaching and popularizing the Runge-Kutta in the educational context. Furthermore, some areas, such as environmental sciences and... Quantitative social sciences still make little use of this resource, indicating opportunities for Methodological expansion and deepening.

Finally, this study contributes to the systematization of knowledge about the use of The Runge-Kutta method for solving ODEs, while also pointing out possible solutions. promising for future research, both in technical improvement and diversification. of its applications. It is believed that this review can serve as a basis for new work. academics, as well as assisting researchers, teachers, and students interested in Mathematical modeling and numerical methods.

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