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# The golden ratio (φ): concepts, applications, and controversies

*A razão áurea (φ): conceitos, aplicações e controvérsias*

*La razón áurea (φ): conceptos, aplicaciones y controversias*

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**Abstract:**
For centuries, the golden ratio has fascinated humanity because it is frequently associated with patterns found in nature, works of art, and architectural structures. Known for expressing mathematical beauty, the golden ratio has been used as a tool for understanding aesthetic patterns. Numbers can help explain the perception of harmony, making mathematics more accessible to readers while preserving the rigor of mathematical concepts. This article presents the mathematical definition of the golden ratio, its historical development, its relationship with the Fibonacci sequence, and its presence in works of art, Classical architectural structures, and natural elements. In addition, it critically examines examples traditionally associated with the golden ratio, highlighting that not all of them are supported by sufficient historical or scientific evidence, and that some remain controversial in the specialized literature. Finally, the article discusses how the golden ratio can be used as a didactic resource in mathematics education, aiming to bring readers closer to this important concept without compromising scientific rigor.

**Keywords:**
Golden ratio; Fibonacci sequence; Mathematics education.

**Resumo:**
Há séculos, a razão áurea deixa a humanidade fascinada, porque ela é associada frequentemente a padrões encontrados na natureza, em obras de arte e construções arquitetônicas. Conhecida por expressar a beleza matemática, a razão áurea é utilizada como ferramenta para compreender padrões estéticos. Os números podem ajudar a explicar a percepção de harmonia, aproximando o leitor da matemática de forma acessível, sem perder a formalidade dos conceitos matemáticos. Este artigo aborda sua definição matemática, seu desenvolvimento histórico, sua relação com a sequência de Fibonacci e sua presença em obras de arte, arquitetônicas da antiguidade clássica e em elementos da natureza. Além disso, discute de forma crítica os exemplos tradicionalmente associados à razão áurea, destacando que nem todos possuem comprovação histórica ou científica suficiente, sendo alguns objetos de controvérsia na literatura especializada. Por fim, aborda como utilizá-la como recurso didático no ensino de Matemática, buscando aproximar o leitor desse importante conceito sem abrir mão do rigor científico.

**Palavras-chave:**
Razão áurea; Sequência de Fibonacci; Educação matemática.

**Resumen:**
Desde hace siglos, la razón áurea fascina a la humanidad porque se asocia frecuentemente con patrones encontrados en la naturaleza, obras de arte y construcciones arquitectónicas. Conocida por expresar la belleza matemática, se utiliza como herramienta para comprender patrones estéticos. Los números pueden ayudar a explicar la percepción de armonía, acercando al lector a las matemáticas de forma accesible sin perder la formalidad de los conceptos matemáticos. Este artículo aborda su definición matemática, su desarrollo histórico, su relación con la sucesión de Fibonacci y su presencia en obras de arte, construcciones arquitectónicas de la

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Antigüedad clásica y elementos de la naturaleza. Además, discute críticamente los ejemplos tradicionalmente asociados a la razón áurea, destacando que no todos cuentan con suficiente comprobación histórica o científica y que algunos son objeto de controversia en la literatura especializada. Finalmente, aborda su uso como recurso didáctico en la enseñanza de las matemáticas, buscando acercar al lector a este importante concepto sin renunciar al rigor científico.

**Palabras clave:**
Razón áurea; Sucesión de Fibonacci; Educación matemática.

## 1. Introduction

When coming across a shell on the beach, who has never stopped to admire its perfect spiral? Is the golden ratio present in this natural element, or is this merely a mistaken association with mathematical patterns that may not exist with such precision? The golden ratio is an ancient, elegant mathematical relationship associated with perfection. This ratio is an irrational constant that became known as “divine.” This fact leads us to question whether it is present everywhere or whether we are seeing patterns where they do not exist so precisely.

Despite the golden ratio's popularity, many examples traditionally cited in art, architecture, and nature are debated by mathematicians, art historians, and other researchers. In numerous cases, scholars identified the proportions retrospectively, without historical documentation indicating that architects or artists consciously used this mathematical relationship in their designs. It therefore becomes important to distinguish historically documented evidence from retrospective interpretations, examining the golden ratio with scientific rigor and critical judgment.

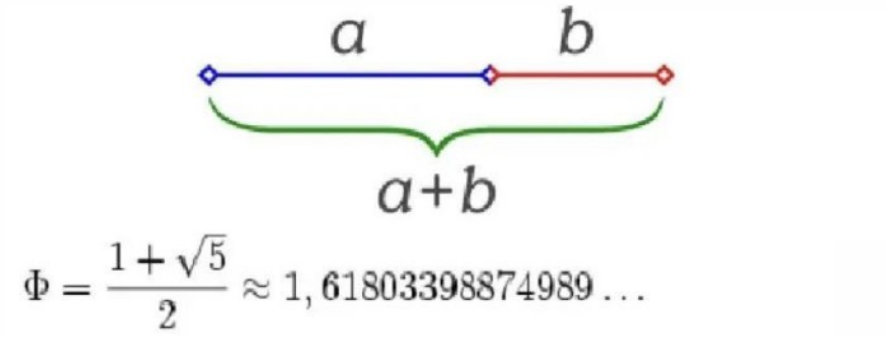
In this article, we study the golden ratio from different perspectives. First, we present its mathematical definition and how its approximate value is obtained. Next comes a historical study from Antiquity to the present. Regarding the Renaissance, we discuss the golden proportion in Leonardo da Vinci's works and in the Fibonacci sequence. Finally, we study the golden ratio as a tool for understanding aesthetic patterns, while warning against excessive interpretations of its presence, and examine its applicability in mathematics classes. In this way, we aim to bring readers closer to mathematics, making it accessible without overlooking the rigor that mathematics demands.

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## 2. The golden ratio, golden angle, and golden rectangle

Given a segment AB, let us divide it into two parts: a longer part and a shorter part, denoted by “a” and “b,” respectively. Imposing the condition that the ratio of the longer part to the shorter part equals the ratio of the entire segment to the longer part, we have

$$(a + b)/a = a/b = \varphi, \quad a > b > 0.$$




$$\Phi = \frac{1+\sqrt{5}}{2} \approx 1,61803398874989\ldots$$

Rewriting the equation with the unknown φ, we obtain

$$(a + b)/a = 1 + 1/\varphi = \varphi \Leftrightarrow \varphi + 1 = \varphi^2 \Leftrightarrow \varphi^2 - \varphi - 1 = 0$$

Solving the quadratic equation,

$$\varphi^2 - \varphi - 1 = 0$$
$$\Delta = b^2 - 4ac = (-1)^2 - 4(1)(-1) = 1 + 4 = 5$$
$$\varphi = (-b \pm \sqrt{\Delta})/(2a) = (1 \pm \sqrt{5})/2$$

Since a > b > 0, we have φ > 0, that is,

$$\varphi = (1 + \sqrt{5})/2 \approx 1.61803398874989\ldots$$

The irrational number obtained is called the golden ratio. Its mathematical beauty lies not only in its numerical value, but also in the simplicity with which it emerges from an elementary geometric construction and the frequency with which it appears in seemingly unrelated contexts.

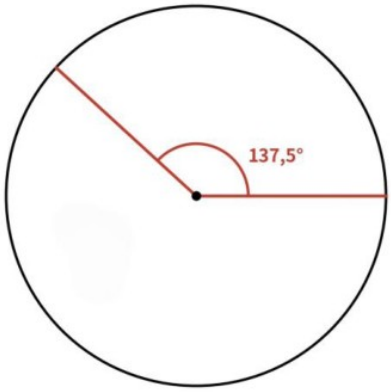
The golden angle is obtained by dividing a circumference so that the ratio of the larger arc to the smaller arc equals the golden ratio. Mathematically, let x be the fraction of the circumference corresponding to the measure of the smaller arc.

Then,

$$(1 - x)/x = \varphi \Leftrightarrow \varphi x = 1 - x \Leftrightarrow \varphi x + x = 1$$
$$\Leftrightarrow x(\varphi + 1) = 1 \Leftrightarrow x = 1/(\varphi + 1)$$

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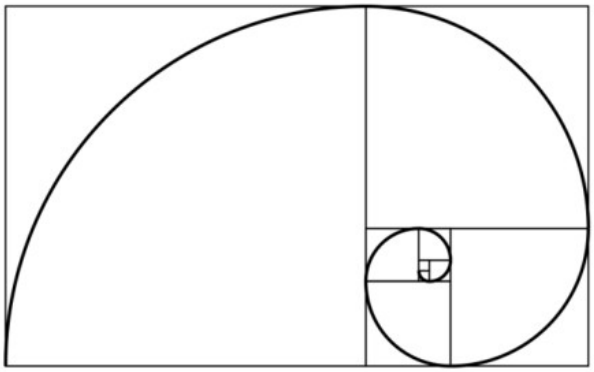
As seen previously, $\varphi + 1 = \varphi^2$, and therefore $x = 1/\varphi^2$. Thus, the golden angle is given by $\theta = 360°/\varphi^2 \approx 137.5°$. This value is known as the golden angle and frequently appears in growth patterns, particularly in phyllotaxis.




A golden rectangle is a rectangle whose longer-to-shorter side ratio equals the golden ratio. One of its most interesting properties is self-similarity. If we remove a square whose side equals the rectangle's shorter side, the remaining rectangle is also a golden rectangle. This process can be repeated indefinitely, generating a sequence of increasingly smaller golden rectangles.

From this successive construction of squares, a smooth curve can be drawn that approximates a logarithmic spiral, known as the golden spiral. This spiral frequently appears in natural patterns and is widely used in artistic compositions and design.

The golden rectangle can be constructed geometrically from a square using a ruler and compass, reinforcing its essentially geometric nature and making it an excellent teaching resource for studying the golden proportion.



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## 3. Historical origins and development

In Ancient Greece, around the sixth century BCE, the Pythagoreans sought to understand the universe through numerical relationships, which sparked interest in the golden ratio. They believed that the universe's harmony was linked to perfect mathematical proportions. There are no records that the Pythagoreans explicitly formulated the golden ratio, but their contributions to the study of numbers and geometry laid the groundwork for the concept's subsequent development.

Around 300 BCE, the mathematician Euclid presented the first definition of the golden ratio in his work The Elements. The definition consisted of dividing a segment into a longer and a shorter part so that the ratio of the longer part to the shorter part equaled the ratio of the whole segment to the longer part. This geometric definition remained a reference for centuries.

During the Middle Ages, the golden ratio did not receive much attention, remaining present only in geometric studies through Euclid's work. It was not until the thirteenth century that the golden ratio again attracted attention in Europe. Through the famous rabbit problem, the mathematician Leonardo Fibonacci introduced the sequence that now bears his name. At the time, Fibonacci did not realize the connection between his sequence and the golden ratio. This relationship was recognized only in the nineteenth century, when it was shown that the limit of the ratio of two consecutive Fibonacci terms converges exactly to the golden ratio.

During the Renaissance, study of the golden ratio reached its peak with the publication of Divina Proportione in 1509 by the mathematician Luca Pacioli, illustrated by Leonardo da Vinci. The work associated the golden ratio with perfection and divine harmony, helping popularize the term “divine proportion.” Leonardo da Vinci applied these principles in several of his works and studies of the proportions of the human body.

In the seventeenth century, the astronomer Johannes Kepler became interested in the golden ratio. In his studies of the solar system, he associated it with the universe's harmony, stating that it represented one of the “jewels of geometry.” From the nineteenth century onward, and especially in the twentieth century, the golden ratio began to be explored in areas such as architecture, design, and art.

Today, study of the golden ratio continues both mathematically and in its aesthetic and scientific applications. Its historical trajectory shows how a simple geometric concept has passed through different eras and cultures, remaining relevant to this day.

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## 4. The Fibonacci sequence and its relationship with the golden proportion

One of the most significant events in the history of the golden ratio occurred in the thirteenth century, with publication of the sequence now named after the mathematician Leonardo Fibonacci. Presenting the solution to the famous rabbit problem, Fibonacci obtained the sequence 1, 1, 2, 3, 5, 8, 13, 21, .... When we calculate the ratio of a term to its preceding term, the result approaches the golden ratio ever more closely. Fibonacci did not recognize this relationship at the time. Only in the nineteenth century, through the work of mathematicians such as Édouard Lucas, did the close connection between the Fibonacci sequence and the golden ratio become clear.



## 5. The golden ratio in nature

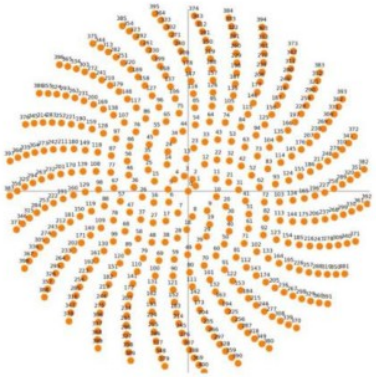
The presence of the golden ratio in nature can be considered extraordinary and, at the same time, one of the most discussed aspects of this mathematical constant. The golden ratio is linked to growth patterns in living beings, particularly those associated with spirals and the distribution of leaves, seeds, and petals around a stem or center. New structures emerge at angles close to the golden angle, allowing efficient distribution, avoiding overlap, and maximizing the use of sunlight, which is essential for plants. In sunflowers, seeds organize into spirals that approximate the Fibonacci sequence and golden angle.

Besides sunflowers, the golden angle can also be observed in the leaf arrangement of various plant species, such as Aloe polyphylla, promoting light capture and a more efficient distribution of structures. In daisies, the number of petals follows the Fibonacci numbers, revealing the relationship between growth patterns and the golden ratio. Other well-documented

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examples include pineapples and pinecones, whose segments and scales form spirals counted in consecutive terms of the Fibonacci sequence.





## 6. The golden ratio in art and architecture

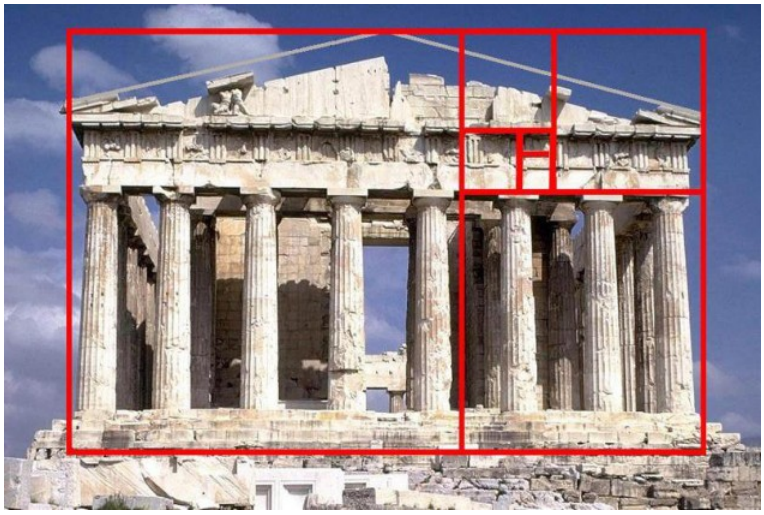
Throughout history, the golden ratio has frequently been associated with art and architecture. In architecture, the Parthenon is one of the most frequently cited examples associated with the golden ratio, since the temple's overall proportions approximate the golden rectangle, contributing to the sense of harmony and beauty the building conveys to this day. However, there are no historical records that it was built according to the golden ratio. Similarly, although some Ancient Egyptian pyramids have dimensions approaching the golden proportion, researchers do not agree on whether this application was intentional.

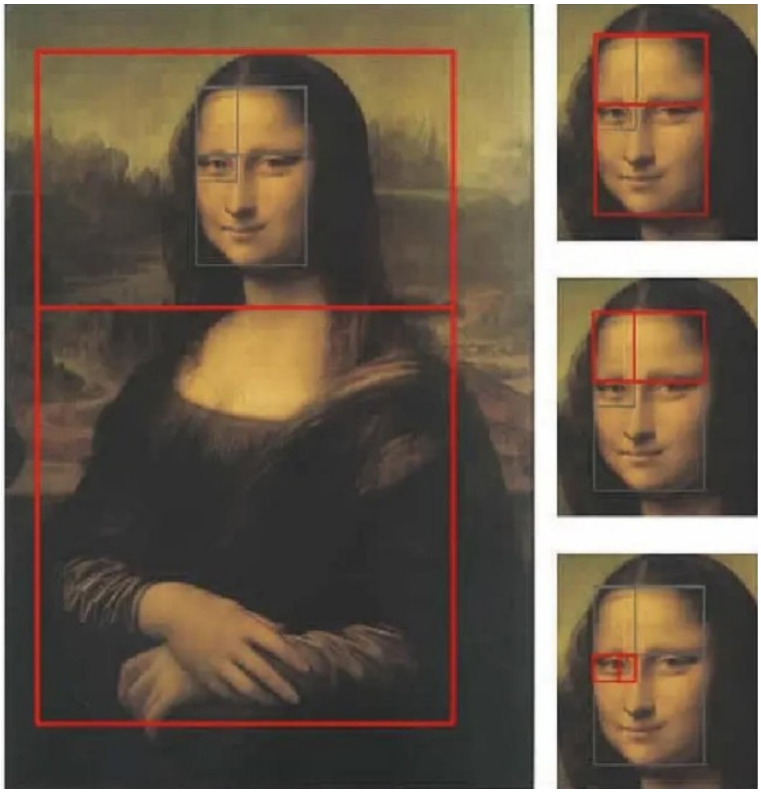
During the medieval period, the golden ratio was also associated with some great Gothic cathedrals, particularly Notre-Dame Cathedral in Paris. Although various scholars identify proportions approaching the golden ratio in its façade and other structural elements, no historical documentation proves that medieval builders intentionally used this proportion.

During the Renaissance, Leonardo da Vinci studied the golden ratio, applying it in various works and artistic compositions. One of his most famous works is the Vitruvian Man, in which Leonardo depicted the ideal proportions of the human body. We cannot state whether the drawing strictly uses the golden ratio, but Leonardo's inspiration from mathematical principles of harmony and proportion is undeniable.

Today, the golden ratio is used in graphic design, typography, and digital interface design. Many designers rely on the golden rectangle and golden spiral to balance visual elements and make them aesthetically pleasing, even unconsciously. The presence of the golden proportion in art and architecture was not always intentional, since in many cases it arose

naturally from the pursuit of visual balance or the technical limitations of the time. It is therefore essential to examine each work carefully, avoiding exaggerated interpretations that the golden ratio is everywhere.





## 7. Critical discussion: science, myth, or a useful tool?

The widespread presentation of the golden ratio in books, documentaries, and other popular-science materials has helped popularize the perception that this proportion occurs in countless contexts, such as works of art, historical monuments, natural elements, and even human beauty. However, not all traditionally cited examples have sufficient historical or

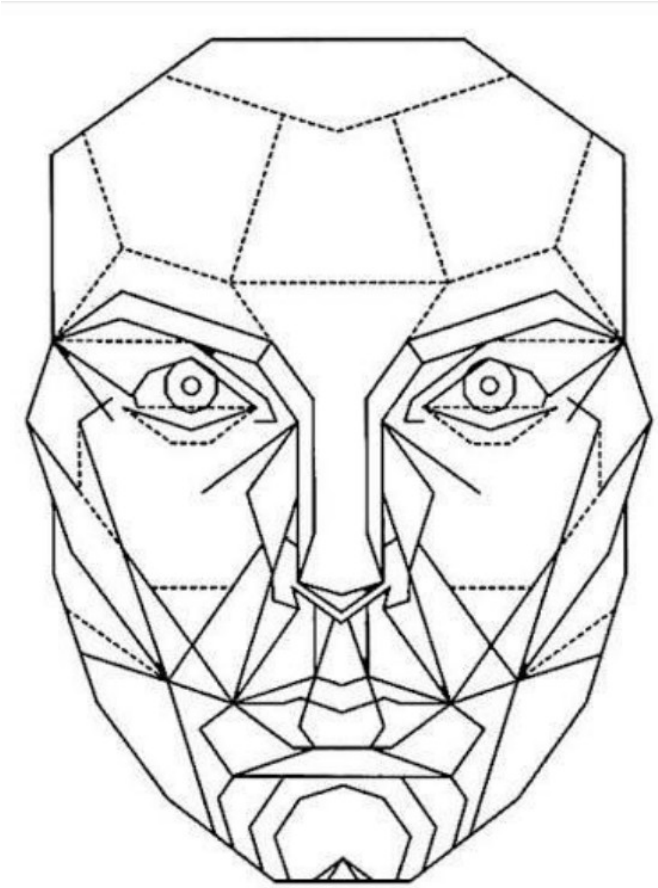
scientific support. This reinforces the need to analyze such associations critically, distinguishing documented evidence from retrospective interpretations.

Although the golden ratio is connected to mathematical elegance and appears in various areas, the concept is often associated with exaggerated interpretations and, in some cases, pseudoscience. Some authors and aesthetics professionals claim to find the golden ratio in works of art, human features, and natural elements. They often adjust measurements or conveniently select data so that their conclusions fit the golden ratio. This numerical juggling compromises the credibility of analyses of its presence. Discussions of facial beauty provide an example. In recent years, the idea of an “ideal face” based on the golden ratio has spread. This “ideal face” is represented by Stephen Marquardt's Mask, considered a tool for classifying a face as attractive. Faces deemed most attractive follow proportions close to φ. Although popular in aesthetics, the mask is questioned by the scientific community. Critics argue that no evidence supports a universal model for classifying beauty, since beauty standards vary significantly across populations, weakening the hypothesis of a universal mathematical standard.

Similar controversies surround classic examples often associated with the golden ratio, such as the Parthenon, the Mona Lisa, and some natural formations. In many cases, no historical records establish intentional use of this proportion, and different measurement methods can lead to different conclusions. Although these examples remain widely publicized, their use as evidence of the golden ratio's presence therefore remains a matter of debate among researchers.

As Mario Livio highlights in Razão Áurea (2008), although the golden proportion has undeniable mathematical beauty, caution is needed to avoid turning it into a kind of “master key” capable of explaining every form of harmony and beauty. Livio observes that many supposed manifestations of the golden ratio in art and nature are actually approximations or the result of forced interpretations.

Nevertheless, the golden ratio has real, documented applications and is consciously used in areas such as graphic design, architecture, and computer graphics, among others. It is recognized as a useful, elegant mathematical tool, provided it is applied rigorously without claiming to associate it with every aesthetic and natural phenomenon. Adopting a critical stance does not mean devaluing it, despite its exaggerated and decontextualized use. Mathematics can thus contribute to understanding patterns of beauty and harmony, but not all beauty can be deduced from a mathematical formula.



## 8. The golden ratio in elementary and lower secondary education: teaching possibilities

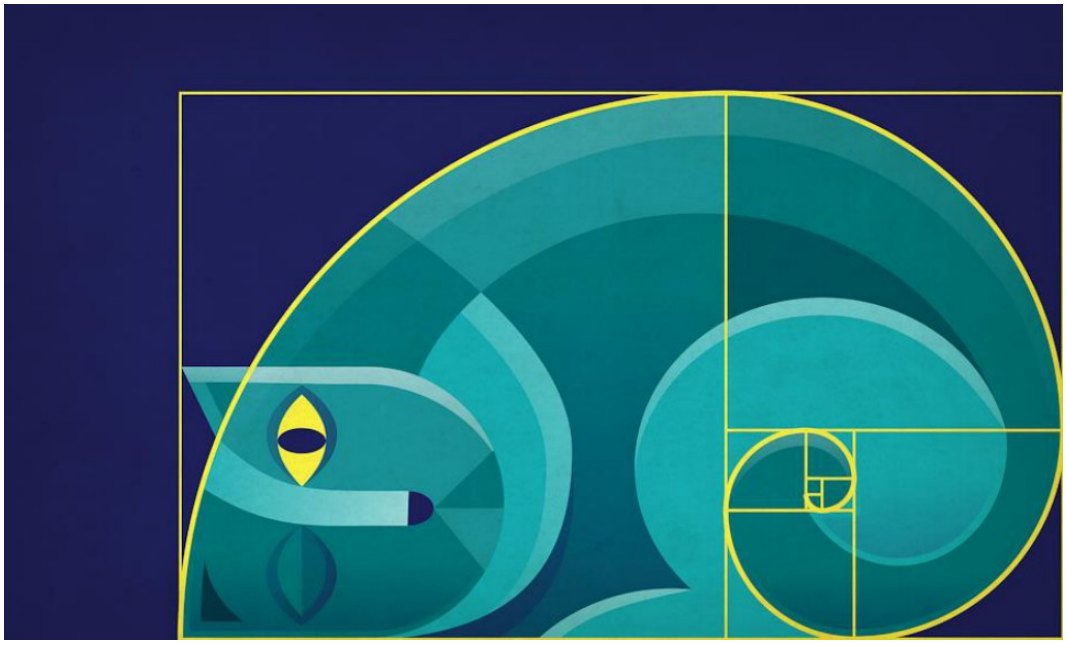
Although irrational numbers are included among the skills prescribed by Brazil's National Common Curricular Base (BNCC) for the final years of elementary and lower secondary education, classroom treatment is generally superficial and has little significance for students. The topic is usually limited to presenting the characteristics of irrational numbers through classic examples such as π and the square root of 2. Students thus fail to develop a deeper understanding of the concept or how it appears in everyday situations. In this context, the golden ratio becomes an important ally in making the study of irrational numbers more concrete, visual, and motivating. As an irrational constant arising from a simple geometric construction and appearing in various contexts, it allows students to encounter an irrational number more tangibly.

Within the mathematics curriculum, work with the golden ratio allows the integration of various knowledge areas prescribed by the BNCC, including notions of proportionality, quantities and measurement, plane geometry, sequences and patterns, and the construction of

concepts related to real numbers. It also promotes socio-emotional and cognitive competencies such as critical thinking, reasoning, and the ability to establish relationships between mathematics and everyday life.

Several activities can be developed in elementary and lower secondary education to approach the golden ratio in an engaging way. Asking students to construct a golden rectangle and observe the spiral formed by squares whose side lengths follow the Fibonacci sequence helps dispel the idea that irrational numbers are merely numbers with infinitely many decimal places that do not appear in everyday situations. Another possibility is to analyze images of nature (sunflowers, shells, and pinecones) to investigate the presence of the golden ratio, encouraging students to observe harmonious shapes through measurements and calculations of ratios. Since the golden ratio appears in various contexts, interdisciplinary activities are also possible. In Art classes, students can create visual compositions using the golden rectangle and analyze artworks displaying the golden ratio. In History classes, the Renaissance context can be explored with a focus on Leonardo da Vinci's works.

While developing these activities, teachers must adopt a critical stance, since many teaching materials and online resources present the golden ratio in an exaggerated or pseudoscientific manner, claiming its presence in virtually every natural and artistic phenomenon. Teachers should alert students to the risk of exaggerated interpretations, distinguishing what can be demonstrated mathematically from what represents merely an approximation or visual coincidence.



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## 9. Conclusion

The golden ratio remains one of mathematics' most fascinating concepts because it establishes a relationship between numerical rigor and the perception of beauty and harmony. In this article, we sought to understand its mathematical definition, its historical development, its relationship with the Fibonacci sequence, its presence in nature, and its influence on art and architecture.

As Mario Livio emphasizes in Razão Áurea (2008), care is needed to avoid turning it into a universal explanation for every form of beauty, avoiding so-called numerical juggling. Although many exaggerated interpretations exist, the golden ratio does indeed have real and relevant applications when used rigorously. It has also proved a promising teaching resource for elementary and lower secondary education, bringing students closer to irrational numbers in a more concrete and meaningful way by connecting an abstract concept to everyday situations.

The golden ratio therefore proves to be an elegant, useful mathematical tool, provided it is understood without exaggeration and with critical responsibility. Mathematics is not only an abstract language but also a means of understanding the order and beauty of the world around us.

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